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Adaptive Control System Design for Two-Wheeled Robot Stabilization

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***Abstract*—A design of two-wheeled robot (TWR) stabilization system using model reference adaptive control (MRAC) method is presented. A TWR is a statically-unstable non-linear system that requires an active stabilization system to operate. The active stabilization system is a state feedback control system. MRAC is applied to design the TWR active stabilization system. MRAC is an adaptive control scheme which has an adaptation mechanism to make a plant as a control object to behave following a reference model. Performance of the designed stabilization system is demonstrated through computer simulation. The simulation results show that the designed system is able to stabilize the TWR and even when some disturbances are presented .**

Keywords: Two-wheeled robot, self-balancing robot, adaptive control, model reference adaptive control, disturbance rejection.

I. INTRODUCTION

A two-wheeled robot (TWR) is one of the favourite plants in control system study. TWR price is quite cheap but provides quite challenge problems. One of them is stability problem. The TWR is a mobile robot is basically constructed by a robot body that is supported by two wheels. Each wheels is driven by a DC motor such that the TWR is in the class of differential-drive mobile robot. Differential-drive mobile robots have high maneuver capability and moreover, the TWR is only supported by two wheels that increase the maneuver capability. The TWR is promising a high maneuver capability but the two-wheel support makes the TWR is not able to stand by itself. The TWR is a statically unstable system.

An active stabilization system is required for the TWR to stand such that it can be operated and applied for many purposes. The active stabilization system is a states feedback control system. Structure of a control system consists of a plant, a controller, actuators, and sensors. The plant is an object to be controlled, for this case is the TWR. The controller is to determine a control command to stabilize the TWR. The control command is executed by the actuators. The DC motors driving the wheels are applied as the actuators. The actuators execute the control command by generating a control torque to stabilize the robot. The generated control torque has to be exactly in the same amount as required. Incorrect control torque will not able to stabilize the robot. The controller determines the control command based on a control law and states feedback. The states feedback is provided by sensors which measure the robot states. An inertial measurement unit (IMU) which consists of rate gyros and accelerometers is commonly applied as sensor in TWR to measure the robot attitude. Since the sensor and actuator are already provided, the remaining part to build a TWR stabilization system is the controller. A controller has a control law that is an algorithm to calculate the control command. The control law is obtained through a control design process.

A control law can be designed using the available control theories, for example model-based control methods. Control design using model-based control method requires a plant dynamics model. The plant dynamics model is mathematics equations representing the plant dynamics. The model can be obtained through system modeling and system identification. Unfortunately, system identification of an unstable system is quite difficult. System modeling is carried out by applying relevant physical laws and using some assumptions for simplification. TWR system modeling by applying the Newton's laws of motion has been presented in [1], [2], while TWR system modeling by using Euler-Lagrange method has been presented in [3], [4]. Both system modeling approaches of the TWR result in a non-linear mathematics equations which show that the TWR is a non-linear dynamics system.

Non-linearity in TWR dynamics provides a challenge work in control system design for the TWR stabilization. Control design of a non-linear system is more difficult than the linear system. Simplify the problem in control design of a non-linear system can be done by is linearization [5]. Linearization is a method to approach a non-linear system by a linear system. By this approach, linear control system design can be done for the linearized system. A design of TWR stabilization system using linear quadratic requlator (LQR) has been presented in [6]. Comparison of TWR stabilization systems designed using LQR and PID control has been presented in [7]. The comparison result shows that both designed systems are able to stabilize the TWR but system designed using LQR has better performance than the system designed using PID control. LQR is a full states feedback control such that measurement of all states is required. For a case where some of the states are unmeasured, observer can be applied to estimate the unmeasured states. A study on TWR stabilization using LQR and utilizing estimated states as the states feedback has been presented in [8]. The presented studies show that a controller of TWR stabilization system can designed in straight forward manner by applying a linear control method for the linearized TWR system. Linerization provides a simple way to design the controller of a non-linear system including TWR. However, the resulted TWR stabilization system works only for a limited region around an equilibrium point that is used as the linearization base. Therefore, the TWR is stabilized in certain operating area but not the whole operating area. In order to stabilize the whole TWR operating area, the TWR stabilization system has to be designed using a non-linear control method. Control design using non-linear control method can not be done in straight forward manner as using linear control method. This makes control design using non-linear control method is more difficult. Several studies on applying non-linear control methods for TWR stabilization have been presented and the results show that the whole TWR operating area are stabilized [2], [9]–[11].

Stabilizing the whole operating area may require powerful motors that are quite costly. In practical application, it may not need to stabilize the whole operating but enough to stabilize in a certain region. Another important thing to be considered in design a TWR stabilization system is capability to withstand disturbance. Disturbance is always present in real world. Adaptive control system promises an adaptation capability to maintain control system performance in any situations, including when disturbances are present [12]. The adaptation provides a mechanism to adjust control gains such that the control system performance is maintained.

This paper presents a design of TWR stabilization system using adaptive control method, particularly model reference adaptive control (MRAC). The resulted TWR stabilization system is expected to maintain the TWR stability even though disturbances are present. Presentation of the paper is organized as follows. Introduction is given in Section I. Section II describes a TWR model. It will be shown that the TWR is a non-linear dynamics system. Section III describes system states representation of the TWR and linearization of the system states. Design of TWR stabilization system using adaptive control method is presented in Section IV. Performance of the designed system is then evaluated through computer simulations and is presented in Section V. Finally, Section VI concludes the work.

II. Modeling of Two Wheels Robot

A two-wheels robot (TWR) is basically constructed by a robot body and two-wheels. The wheels are to support the robot body. Each wheels is driven by a DC motor. Model of the TWR is shown in Figure 1a. The robot body is represented by a linkage where the center of mass is assumed to be located at the middle of the linkage. This study is only concerned in longitudinal dynamics of TWR and assumes that both wheels move in the same motion, such that both wheels move in the same velocity and direction. Therefore, the TWR motion is limited in two degree of freedom (2 DOF), i.e.: one translation and one rotation (pitch motion).

Assume that a TWR is initially at standing position. Presenting a disturbance to the TWR may make the TWR to pitch. Attitude of the TWR is denoted by a pitch angle, θ, which is angle of the robot body with respect to vertical axis. At the pitch position with pitch angle θ as shown in Figure 1a, weight of the robot body gives a moment and makes the robot to fall down. It is the reason why the robot is statically unstable. In order to keep the robot stable which is turning back the robot to the initial attitude, a torque is required to counter the disturbance and the moment due to the weight. This required torque is then called as the control torque.

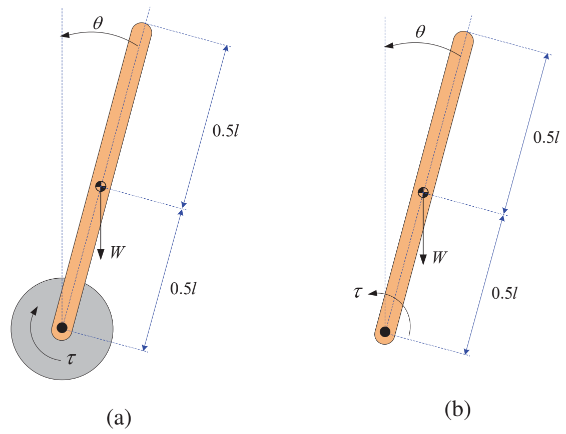

Fig. 1: (a) Two-wheeled robot (TWR) model. (b) Free-body diagram of the TWR model.

While the DC motors are active, they generate torque to rotate the wheels. Friction of the wheel and the floor results in reaction torque to the body. Figure 1b shows free body diagram of the robot. There are two moments working on the robot body, i.e.: moment due to the body weight and moment due to the reaction torque. The reaction torque can be utilized as a control torque to stabilize the robot. Therefore, the DC motors have two functions: driving the wheels and actuator to stabilize the robot.

In order to derive the TWR dynamic, consider Figure 1b and use the wheel axis as the base point to evaluate the working torques on the TWR body. Applying the Newton's second law results in the following dynamic equations:

$$\Sigma M = I\ddot{\theta} \quad (1)$$

$$\tau - \frac{1}{2}mgl\sin\theta = I\ddot{\theta} \quad (2)$$

where M is the working moment on the TWR, I is the TWR-body inertia, θ is the TWR pitch angle, m is the TWR-body mass, g is the gravity acceleration, l is the TWR-body length, $\ddot{\theta}$ is the pitch angular acceleration, and τ is the control torque to stabilize the TWR. By defining $I_r = \frac{1}{2}mgl$ the (2) can be expressed as:

$$I\ddot{\theta} + I_r\sin\theta = \tau. \quad (3)$$

III. System States Representation and Linearization

Define the following system states

$$\begin{aligned} x_1 &= \theta \\ x_2 &= \dot{\theta} \end{aligned} \quad (4)$$

and substituting (4) into (3) such that the TWR dynamic equations can be expressed in a system states equation as follows:

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -\frac{I_r}{I}\sin x_1 + \frac{1}{I}\tau. \end{aligned} \quad (5)$$

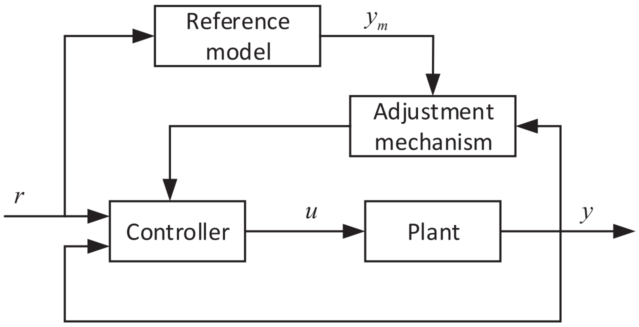

Fig. 2: Block diagram of model reference adaptive control (MRAC) [12].

Simplify (5) by defining constants $k_1 = \frac{I_r}{I}$ and $k_2 = \frac{1}{I}$ and substituting into (5) such that results in:

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -k_1 \sin x_1 + k_2 \tau. \end{aligned} \tag{6}$$

Equation (6) is the states equation of TWR dynamics system. It is shown that the TWR is a non-linear dynamics system. Linearization to the system is done to simplify in control design process. The system is linearized around the origin ($\theta = 0$, $\dot{\theta} = 0$) and results in:

$$\dot{x} = Ax + Bu, \tag{7}$$

where $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$, $A = \begin{bmatrix} 0 & 1 \\ k_1 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ k_2 \end{bmatrix}$, and $u = \tau$.

IV. Adaptive Control Design

Adaptive control theory discussed in this section is adopted from [12], [13] and described as follows. For a system:

$$\dot{x} = Ax + Bu \tag{8}$$

defined a model reference:

$$\dot{x}_m = A_m x_m + B_m r. \tag{9}$$

Define a control input

$$u = -K_1 x + K_2 r. \tag{10}$$

and substituting (10) into (8) results in:

$$\dot{x} = (A - BK_1)\, x + BK_2 r. \tag{11}$$

Define $A_c = A - BK_1$ and $B_c = BK_2$ such that (11) can be expressed by:

$$\dot{x} = A_c x + B_c r. \tag{12}$$

Goal of this control design is to make the closed-loop system (12) to behave following the reference model (9). In order to achieve the goal, define states error between the closed-loop system and the reference model as follows:

$$e = x - x_m \tag{13}$$

and derivating the states error with respect to time results in:

$$\dot{e} = A_c x - A_m x_m + (B_c - B_m)\, r. \tag{14}$$

Adding and subtracting $A_m x$ into (14) result in:

$$\dot{e} = A_m e + (A_c - A_m)\, x + (B_c - B_m)\, r. \tag{15}$$

Define $\tilde{A} = A_c - A_m$ and $\tilde{B} = B_c - B_m$ such that (15) can be expressed by:

$$\dot{e} = A_m e + \tilde{A}x + \tilde{B}r. \tag{16}$$

In order to make e converge to zero for $t \to \infty$, define a Lyapunov function:

$$V = e^T P e + \text{tr}\left(\tilde{A}^T \tilde{A}\right) + \text{tr}\left(\tilde{B}^T \tilde{B}\right), \tag{17}$$

where operator tr is trace of matrix. Derivating the Lyapunov function with respect to time results in:

$$\begin{aligned} \dot{V} &= \dot{e}^T P e + e^T P \dot{e} + 2\text{tr}\left(\tilde{A}^T \dot{\tilde{A}}\right) + 2\text{tr}\left(\tilde{B}^T \dot{\tilde{B}}\right) \\ &= \left[A_m e + \tilde{A}x + \tilde{B}r\right]^T P e + e^T P \left[A_m e + \tilde{A}x + \tilde{B}r\right] \\ &\quad + 2\text{tr}\left(\tilde{A}^T \dot{\tilde{A}}\right) + 2\text{tr}\left(\tilde{B}^T \dot{\tilde{B}}\right) \\ &= \left[e^T A_m^T + x^T \tilde{A}^T + r^T \tilde{B}^T\right] P e + 2\text{tr}\left(\tilde{A}^T \dot{\tilde{A}}\right) \\ &\quad + e^T P \left[A_m e + \tilde{A}x + \tilde{B}r\right] + 2\text{tr}\left(\tilde{B}^T \dot{\tilde{B}}\right) \\ &= e^T \left(A_m^T P + P A_m\right) e + x^T \tilde{A}^T P e + e^T P \tilde{A} x \\ &\quad + r^T \tilde{B}^T P e + e^T P \tilde{B} r \\ &\quad + 2\text{tr}\left(\tilde{A}^T \dot{\tilde{A}}\right) + 2\text{tr}\left(\tilde{B}^T \dot{\tilde{B}}\right) \end{aligned} \tag{18}$$

Using algebraic Riccati equation, it can be found a matrix Q where:

$$A_m^T P + P A_m = -Q \tag{19}$$

such that (18) becomes:

$$\begin{aligned} \dot{V} &= -e^T Q e + 2x^T \tilde{A}^T P e + 2r^T \tilde{B}^T P e + 2\text{tr}\left(\tilde{A}^T \dot{\tilde{A}}\right) \\ &\quad + 2\text{tr}\left(\tilde{B}^T \dot{\tilde{B}}\right). \end{aligned} \tag{20}$$

Negative semidefiniteness of $\dot{V}$ can be achieved by the following condition:

$$\text{tr}\left(\tilde{A}^T \dot{\tilde{A}}\right) = -x^T \tilde{A}^T P e \tag{21}$$

and

$$\text{tr}\left(\tilde{B}^T \dot{\tilde{B}}\right) = -r^T \tilde{B}^T P e. \tag{22}$$

Since both $x^T \tilde{A}^T P e$ and $r^T \tilde{B}^T P e$ are scalar, we can define:

$$\text{tr}\left(x^T \tilde{A}^T P e\right) = \text{tr}\left(\tilde{A}^T P e x^T\right) = x^T \tilde{A}^T P e \tag{23}$$

$$\text{tr}\left(r^T \tilde{B}^T P e\right) = \text{tr}\left(\tilde{B}^T P e r^T\right) = r^T \tilde{B}^T P e. \tag{24}$$

Therefore, the negative semidefiniteness of $\dot{V}$ can be achieved by the following rules:

$$\dot{K}_1 = (B^T B)^{-1} B^T P e x^T \tag{25}$$

$$\dot{K}_2 = -(B^T B)^{-1} B^T P e r^T \tag{26}$$

Note that $\dot{V}$ is now negative definite. Because K_1, K_2, and e are bounded, using Barbalat's lemma, it is shown that the error goes to zero [12].

TABLE I: SIMULATION PARAMETERS

Parameter	Symbol	Value	Unit
Mass of the rod	m_r	0.1	kg
Length of rod	l	0.2	m
Inertia of the rod	I_r	13×10^{-4}	$kg.m^2$

V. Simulation

Performance of the designed TWR stabilization system using adaptive control method is evaluated through computer simulations. Simulations are purposed to show whether that the designed system is able to stabilize the TWR when disturbances are present. Parameters of the TWR used in the simulation are given in Table I. It is assumed that all system states are measured and measurement noises are neglected. For the simulation, define a reference model as follows

$$A_m = \begin{bmatrix} 0 & 1 \\ -100 & -15 \end{bmatrix}, B_m = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

and matrix Q in (19) as the adaptive control parameter is selected as follows

$$Q = \begin{bmatrix} 10^4 & 0 \\ 0 & 1 \end{bmatrix}.$$

Two simulation scenarios are presented as follows. In first simulation, the TWR is passing a smooth road profile. The TWR position is initially at $\theta = 0°$. At $t = 0.5$ seconds, the robot is disturbed by an impulse torque 0.5 Nm such that the TWR pitches with pitch angle θ. An inertial measurement unit (IMU) is commonly installed on a TWR as a sensor to measure angular position and angular rate. In this case, the sensor is modelled by unity gain. The sensor gives information about the pitch angle and pitch angular rate as states feedback to the controller. The controller use the states feedback to calculate a control torque that is required to return the TWR to the initial position, $\theta = 0°$. Figure 3 shows the simulation results. The disturbance makes the robot to pitch about 20°. The result shows that the designed TWR stabilization system is able to return the TWR to the initial position $\theta = 0°$ such that the TWR is asymptotically stabilized. The required time for stabilization is about 1.2 seconds. The required stabilization time can be modified by adjusting reference model parameter (A_m and B_m) and the adaptation parameter (Q).

In second simulation, the TWR is passing a rough road profile. The simulation scenario is done by modifying the first simulation scenario by adding random torque due to the rough road profile. The random torque has amplitude 0.1 Nm and frequency 1 Hertz. Figure 4 shows the simulation results. The designed TWR stabilization is able to keep the TWR to be stable such that the TWR is not falling down. However, the TWR pitch angle is varying in the range of about $\pm5°$ due to the random torque disturbance is presented during the simulation. Figure 5 shows the TWR system trajectory. It is shown clearly that the TWR system is stable as shown by the bounded system trajectory.

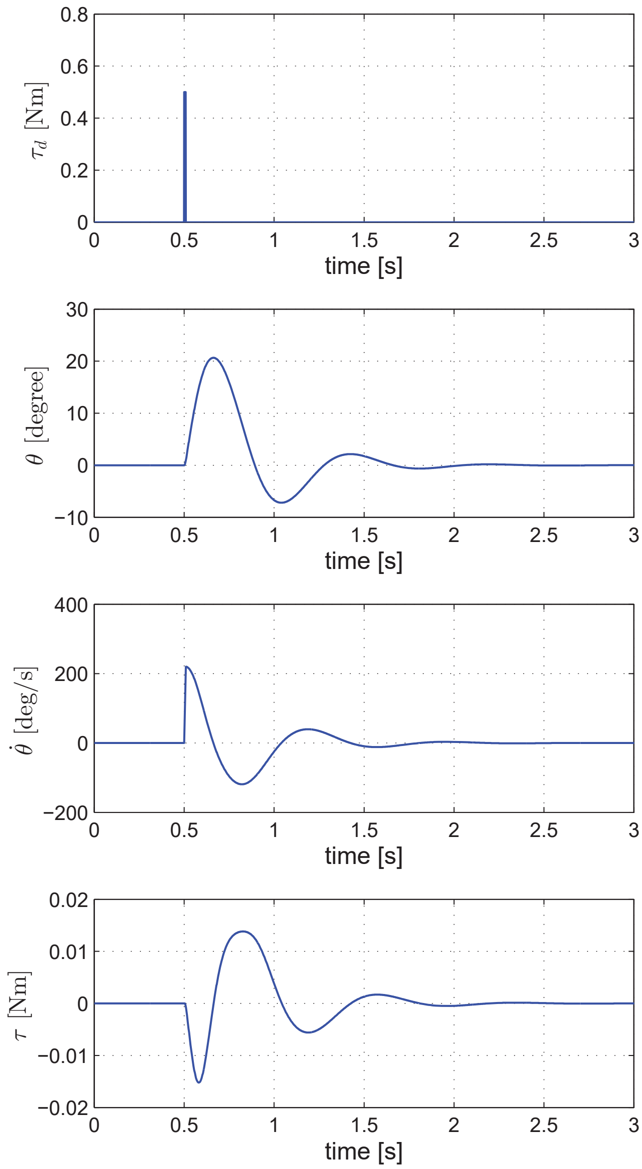

Fig. 3: The TWR response when the robot is disturbed by impulse torque 0.5 Nm.

VI. Conclusion

A design of two-wheeled robot (TWR) stabilization system using model reference adaptive control (MRAC) method has been presented. The TWR stabilization system was designed based on a linearized system. The designed system has adaptation capability to make the closed loop system to behave following a reference model. Through the adaptation capability, the closed loop system shows capability of disturbance rejection, where the closed loop system attempts to maintains the stabilization performance even though a continues disturbance is present. The designed TWR stabilization has both capability of TWR stabilization and disturbance rejection. The designed TWR stabilization is very prospective for practical application.

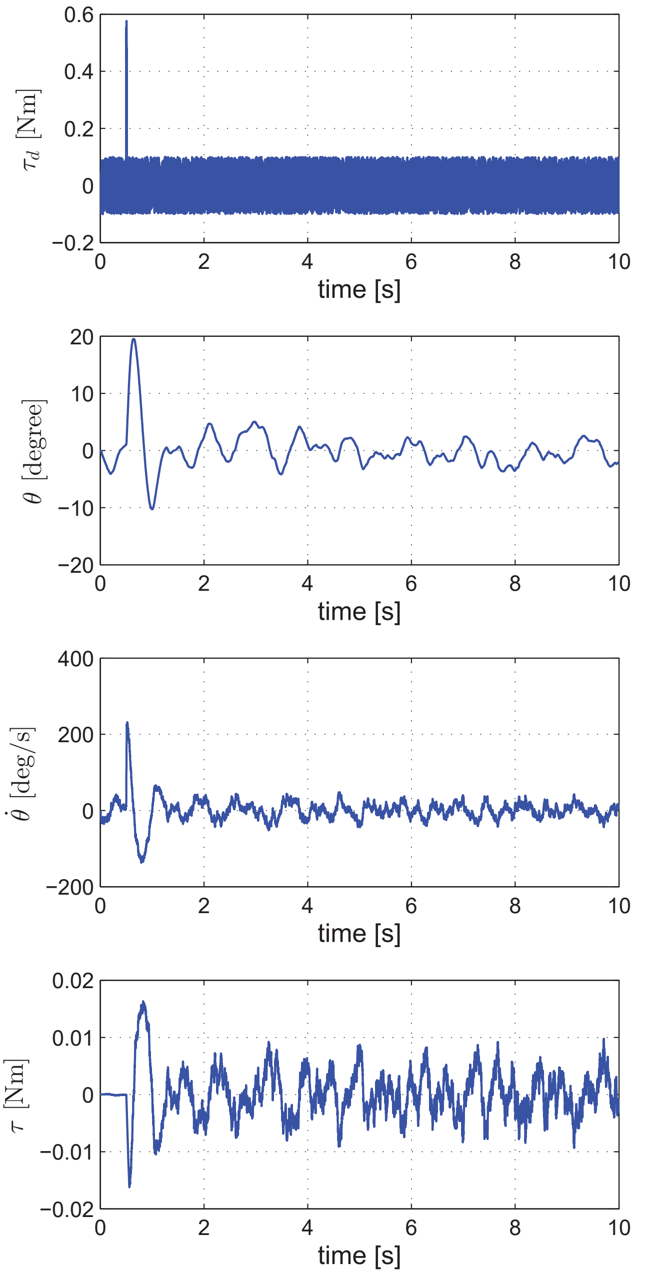

Fig. 4: The TWR response when the robot is disturbed by impulse torque 0.5 Nm and random torque with amplitude 0.1 Nm and frequency 1 Hertz.

VII. Future Works

A TWR is under construction and the designed stabilization system is going to be implemented and tested in real-time.

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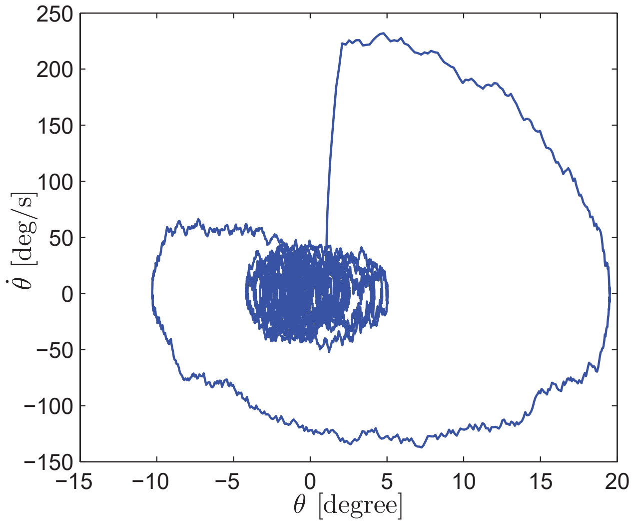

Fig. 5: The TWR system trajectory when the robot is disturbed by impulse torque 0.5 Nm and continuous random torque with amplitude 0.1 Nm and frequency 1 Hertz.

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